A Note on the Optimum Design of I-Section Beams

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Nomenclature

σ_{tu} = ultimate tensile strength, psi
σ_{tp} = proportional limit stress, psi
b = semiwidth of flange, in.
h = semidepth of web, in.
t = thickness, in.
M = applied bending moment, in.-lb
E = modulus of elasticity, psi
η_s = E_s/E
E_s = secant modulus, psi
A = section area, in.^{2}
λ = Lagrangian multiplier

IN Ref. 1 optimum dimensions for minimum weight were obtained based on the assumption that the stress, as well as the strain, is linear on the section; the optimum ratio of depth to flange width of 1.3 was therein obtained. Based on the modulus of rupture it is shown in this paper that the ratio of depth to flange width for minimum weight is 1.46 (see Fig. 1).

Pertinent to ductile materials it is assumed that a rectangular stress distribution exists at ultimate strength. We have the relationship

$$4\sigma_{tu}bth + 2\sigma_{tu}\int_u^h tydy = \sigma_{tu}(4bth + th^2) = M \quad (1)$$

Equating the ultimate strength, based on modulus of rupture, to the local buckling stress of the flange,

$$\sigma_{tu} = \frac{M}{t(4bh + h^2)} = \frac{M}{t(4b^2k + k^2b^2)} = KE\left(\frac{t}{b}\right)^2 \quad (2)$$

where

$$K = (0.615)^2\eta_s \quad (3)$$

The area A of the cross section of the beam may be written

$$A = 2bt(2 + k) \quad (4)$$

where

$$k = h/b \quad (5)$$

Table 1 **Dimensions of optimum I sections and channel section**

	Thickness	Flange width	Web depth	Area
I section(elas)	0.113	1.76	2.29	0.66
I section(plas)	0.152	1.22	1.78	0.64
Channel	0.179	1.39	1.81	0.82

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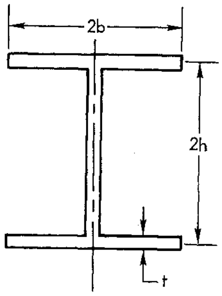

Fig. 1 Beam cross section.

Equation (2) may be simplified to

$$\frac{M}{t(4k + k^2)} = KEt^2 \quad (6)$$

Equation (6) is used to define the constraining relationship

$$\psi(t, k) = M - KEt^3(4k + k^2) \quad (7)$$

The values of k, b, and t, corresponding to the minimum value of A, may be obtained by equating to zero the partial derivatives $\partial u/\partial t$ and $\partial u/\partial k$ of the auxiliary function, defined by Eq. (8), and solving the resulting equations:

$$u = A + \lambda\psi \quad (8)$$

where ψ is a constant,

$$2b(2 + k) - 3KE(4k + k^2)t^2\lambda = 0 \quad (9)$$

Simplifying,

$$2bt - KEt^3(4 + 2k)\lambda = 0 \quad (10)$$

By Eq. (10)

$$\lambda = \frac{2b}{KEt^3(4 + 2k)} \quad (11)$$

Substituting ψ from Eq. (11) into Eq. (9) and solving for k,

$$k = 2[(3)^{1/2} - 1] = 1.46 \quad (12)$$

By Eq. (6)

$$t = \left(\frac{1}{4k + k^2}\right)^{1/3}\left(\frac{M}{KE}\right)^{1/3} = 0.502\left(\frac{M}{KE}\right)^{1/3} \quad (13)$$

and by Eq. (2)

$$b = \left(\frac{KE}{\sigma_{tu}}\right)^{1/2} t \quad (14)$$

Equations 12–14 may be used to obtain the dimension of the optimum section.

Based on an elastic stress distribution the optimum dimensions of a channel section in bending are given by $k = 1.30$, $t = 0.855\ (M/KE)^{1/3}$, $b = (KE/\sigma_{tp})^{1/2}t$. Table 1 compares optimum I sections with elastic and plastic stress distributions, and a channel section with an elastic stress distribution for $M = 36{,}000$ in.-lbs, $K = 0.38$, $E = 10.3\ (10^6)$, $\sigma_{tp} = 65{,}000$ psi, and $\sigma_{tu} = 80{,}000$ psi.

Reference

[1] Krishnan, S. and Shetty, K. V., "On the optimum design of an I-section beam," J. Aerospace Sci. **26**, 599–600 (September 1959).